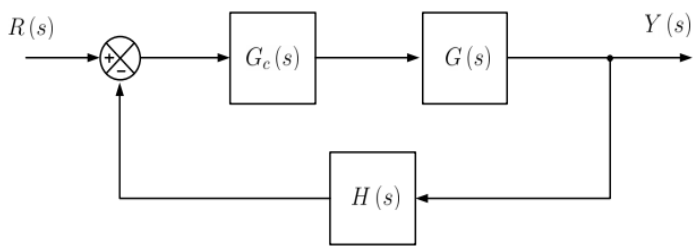
Controller Design using Asymptotic Bode Plots

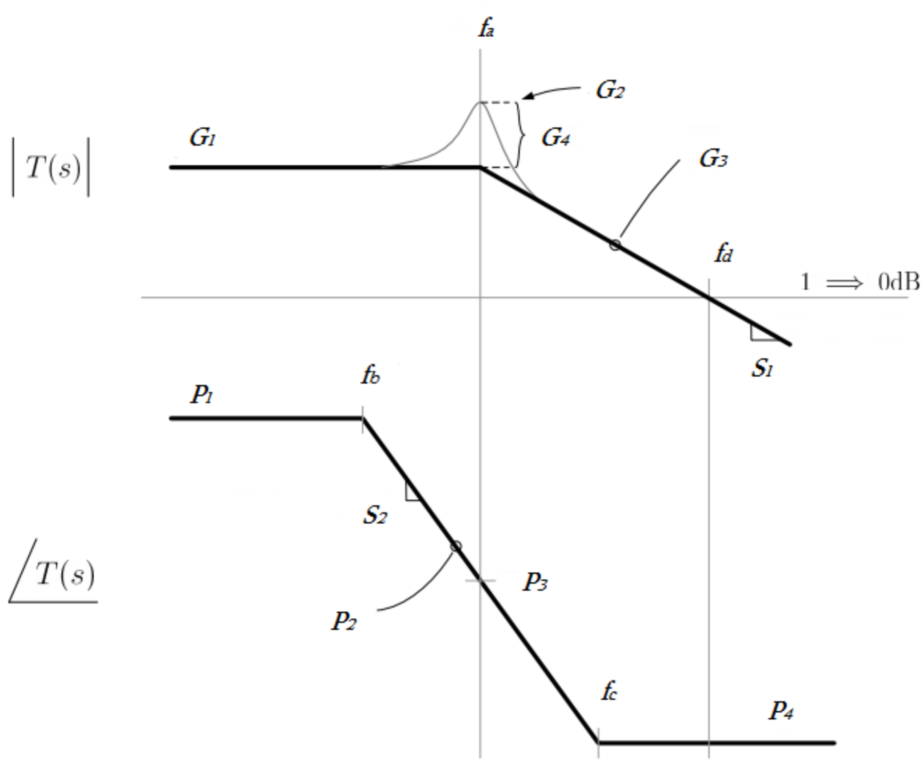
The figure below shows a closed loop control system, where the plant, $G(s)$, is given by

$$G(s) = \frac{G_o}{1 + \frac{s}{Q\omega_o} + \left(\frac{s}{\omega_o}\right)^2}$$

where $G_o = 10,\ Q = 4,\ \omega_o = 2\pi(1000)$. The feedback gain, $H(s) = {}^{1}/_{2}$. Initially the system is uncompensated, so that $G_c(s) = 1$. (Hint: $10^{-1/8} \approx \frac{3}{4}$)



The asymptotic Bode plot of the uncompensated loop gain, $T(s)\big(= G_c(s)G(s)H(s)\big)$, with $G_c(s) = 1$, is:



Determine the following:

Frequencies:

1) f_a
2) f_b
3) f_c

Gains:

1) G_1
2) G_2
3) G_3
4) G_4 (this is a ratio of gains)

Phase values:

1) P_1
2) P_2
3) P_3
4) P_4

Gain and phase slope values:

1) S_1
2) S_2

Determine the phase and gain margins of this uncompenstated system:

1) Phase margin
2) Gain margin

Is the closed loop system stable?

Determine the system type number.

Assuming a unit step input , determine the final value of the output:

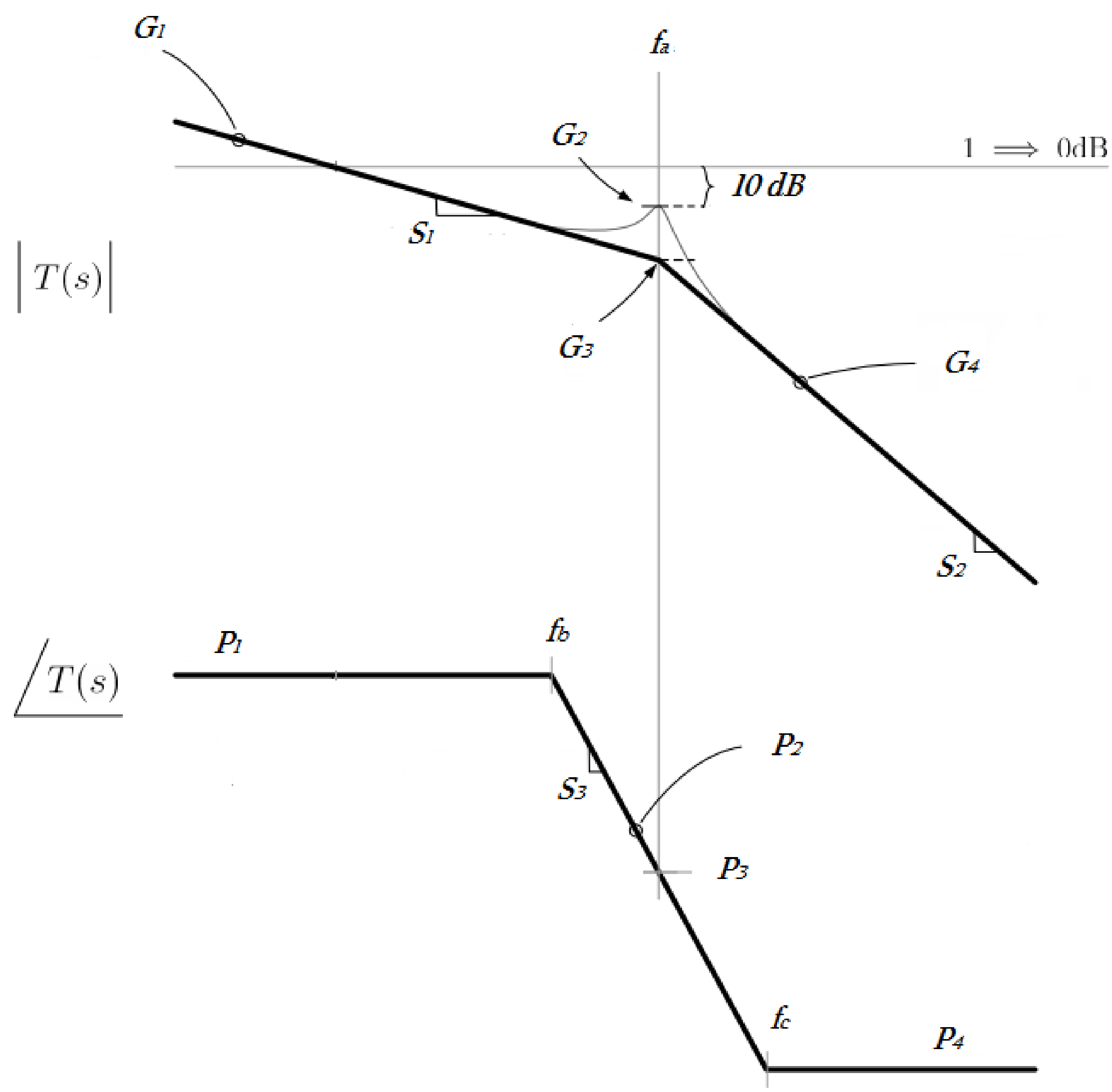
Assuming an ideal gain of 2 $\left(=\frac{1}{H(0)}\right)$, determine the percentage steady state error:

Compensation:

In order to improve the performance of the system an integral compensator is considered:

$$G_c(s) = \frac{\omega_I}{s}$$

To design this compensator the parameter, ω_I, needs to be determined. This will be undertaken with the aid of the asymptotic Bode plot of the compensated loop gain, $T(s)\big(= G_c(s)G(s)H(s)\big)$, with $G_c(s) = \frac{\omega_I}{s}$, shown next:



Determine the following:

Frequencies:

1) f_a
2) f_b
3) f_c

Gains:

1) G_1
2) G_2
3) G_3
4) G_4

Phase values:

1) P_1
2) P_2
3) P_3
4) P_4

Gain and phase slope values:

1) S_1
2) S_2
3) S_3

Using the equations derived so far choose a value of ω_I which places the resonant Q peak at 10dB below the 0dB gain value as shown in the plot above. [Hint: -10 dB → $\frac{1}{\sqrt{10}}$]. Determine the phase and gain margins of the resulting loop gain and their associated frequencies.

1) Phase margin
2) Unity gain frequency (in Hz)
3) Gain margin
4) -180° phase crossover frequency (in Hz)

Is the closed loop system stable?

Determine the system type number.

Assuming a unit step input, determine the final value of the output:

Assuming an ideal gain of 2 $\left(= \frac{1}{H(0)}\right)$, determine the percentage steady state error: